Generic Plaintext Equality and Inequality Proofs

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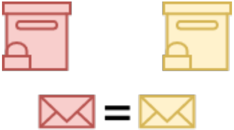
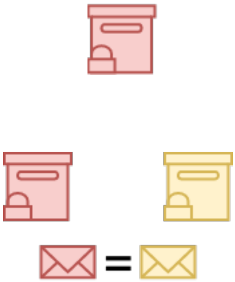
Agenda

1. Motivation
2. Generic Randomizable Encryption
3. Protocols
4. Comparisons for ElGamal
5. Conclusions and Future Work

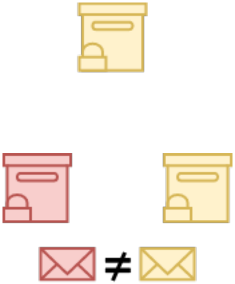


Plaintext Equality (PET)

Motivation

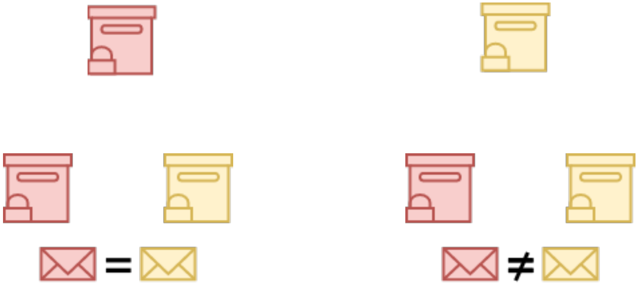


Plaintext Equality (PET)



Plaintext Inequality (PIT)

Motivation



Plaintext Equality (PET) Plaintext Inequality (PIT)

Generic zero knowledge proofs for PET-PIT

Applications

Applications

Voting

Voting

Reputation systems

Applications

Voting

Reputation systems

Cloud applications

Voting

Reputation systems

Cloud applications

Broadcast

Voting

Reputation systems

Cloud applications

Broadcast

Storage

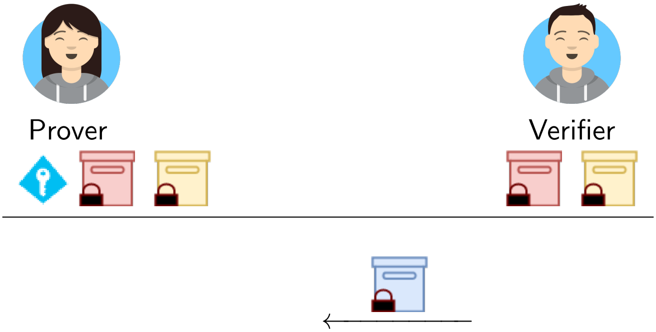
Warm Up Example

Prover

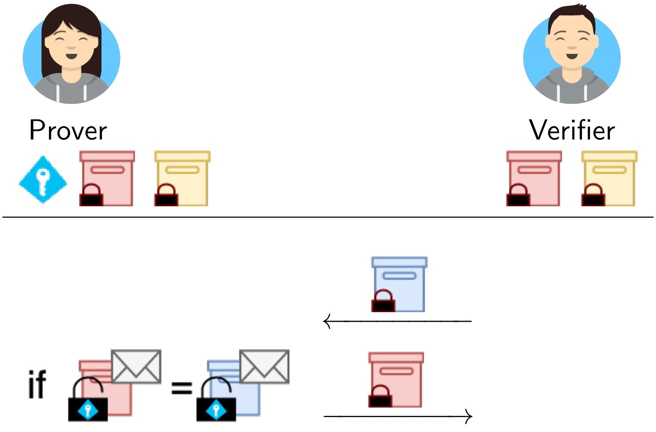
Verifier

Warm Up Example

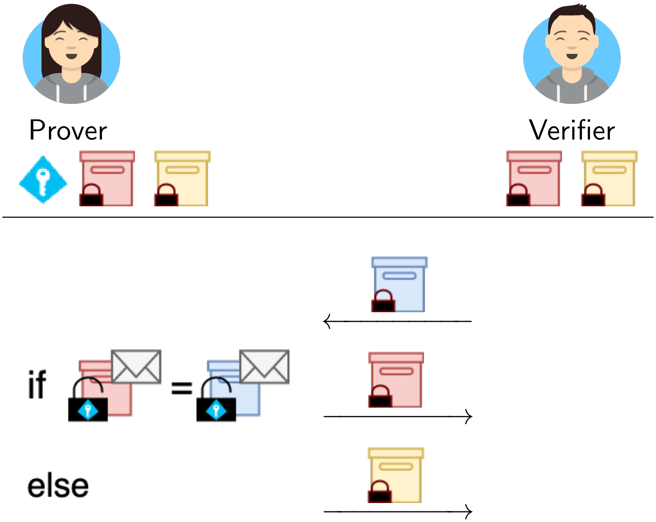
Warm Up Example



Warm Up Example



Warm Up Example



Agenda

Generic Randomizable Encryption

- Ciphertexts (Rand),

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- Ciphertexts (Rand), messages (MsgRand), encryption keys (KeyRand)

- Ciphertexts (Rand), messages (MsgRand), encryption keys (KeyRand) and combinations

Generic Randomizable Encryption

- Ciphertexts (Rand), messages (MsgRand), encryption keys (KeyRand) and combinations
- Formal definitions: randomizabily and strong randomizability, message-randomizability, key-randomizability and random coin decryption (RCD)

Generic Randomizable Encryption

- Ciphertexts (Rand), messages (MsgRand), encryption keys (KeyRand) and combinations
- Formal definitions: randomizabily and strong randomizability, message-randomizability, key-randomizability and random coin decryption (RCD)
- Two flavours: computational and perfect

Agenda

Protocols

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- Simple cut-and-choose protocols

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- Completeness, soundness and perfect zero knowledge

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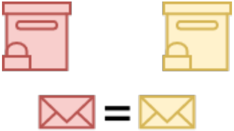
- PET: Rand & MsgRand

Protocols

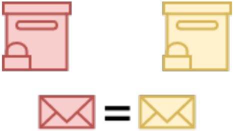
- Simple cut-and-choose protocols
- Completeness, soundness and perfect zero knowledge
- PIT: Rand
- PET: Rand & MsgRand
- Sigma PET's: Rand, MsgRand & (KeyRand $\vee$ RCD)

Protocols



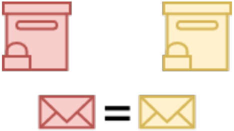
PET

Protocols



PET

- $\mathsf{pk}_0 = \mathsf{pk}_1$ and the prover knows sk_0



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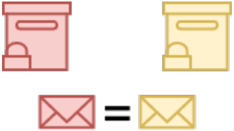
- $\mathsf{pk}_0 = \mathsf{pk}_1$ and the prover knows sk_0
  $\rightsquigarrow$ HPEQ, PEQ

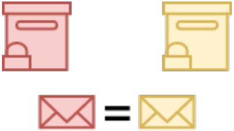
Protocols



PET

- $\mathsf{pk}_0 = \mathsf{pk}_1$ and the prover knows sk_0
  $\rightsquigarrow$ HPEQ, PEQ
- $\mathsf{pk}_0 \neq \mathsf{pk}_1$ and the prover knows sk_0 and sk_1

Protocols



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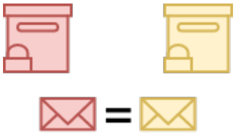
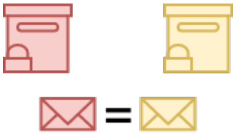
- $pk_0 = pk_1$ and the prover knows sk_0
  $\rightsquigarrow$ HPEQ, PEQ
- $pk_0 \neq pk_1$ and the prover knows sk_0 and sk_1
  $\rightsquigarrow$ MATCHPEQ, SIGPEQ



PET

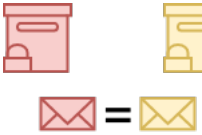
- $pk_0 = pk_1$ and the prover knows sk_0
  $\rightsquigarrow$ HPEQ, PEQ
- $pk_0 \neq pk_1$ and the prover knows sk_0 and sk_1
  $\rightsquigarrow$ MATCHPEQ, SIGPEQ
- $pk_0 \neq pk_1$ and the prover knows r_0 and r_1



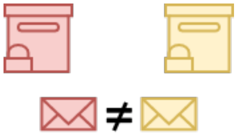
PET

- $\mathsf{pk}_0 = \mathsf{pk}_1$ and the prover knows sk_0
  $\rightsquigarrow$ HPEQ, PEQ
- $\mathsf{pk}_0 \neq \mathsf{pk}_1$ and the prover knows sk_0 and sk_1
  $\rightsquigarrow$ MATCHPEQ, SIGPEQ
- $\mathsf{pk}_0 \neq \mathsf{pk}_1$ and the prover knows r_0 and r_1
  $\rightsquigarrow$ RSPEQ

Protocols



PET



PIT

- $pk_0 = pk_1$ and the prover knows sk_0
  $\rightsquigarrow$ HPEQ, PEQ
- $pk_0 \neq pk_1$ and the prover knows sk_0 and sk_1
  $\rightsquigarrow$ MATCHPEQ, SIGPEQ
- $pk_0 \neq pk_1$ and the prover knows r_0 and r_1
  $\rightsquigarrow$ RSPEQ

- $pk_0 = pk_1$ and the prover knows sk_0
  $\rightsquigarrow$ HPINEQ, PINEQ

Alice $(\mathsf{sk}, \mathsf{pk}, c_0, c_1)$

Bob (pk, c_0, c_1)

HPINEQ

Alice $(\mathsf{sk}, \mathsf{pk}, c_0, c_1)$

Bob (pk, c_0, c_1)

$r \xleftarrow{\$} \mathcal{R};$

HPINEQ

Alice $(\mathsf{sk}, \mathsf{pk}, c_0, c_1)$

Bob (pk, c_0, c_1)

$r \xleftarrow{\$} \mathcal{R};\ b \xleftarrow{\$} \{0, 1\}$

HPINEQ

Alice $(\mathsf{sk}, \mathsf{pk}, c_0, c_1)$

Bob (pk, c_0, c_1)

$r \xleftarrow{\$} \mathcal{R};\ b \xleftarrow{\$} \{0, 1\}$

$c'_b \leftarrow \mathsf{Rand}(c_b, r)$

$\xleftarrow{\quad c'_b \quad}$

if $\mathsf{Dec}_{\mathsf{sk}}(c'_b) = \mathsf{Dec}_{\mathsf{sk}}(c_0)$

Alice $(\mathsf{sk}, \mathsf{pk}, c_0, c_1)$

Bob (pk, c_0, c_1)

Alice		Bob
		$r \xleftarrow{\$} \mathcal{R};\ b \xleftarrow{\$} \{0,1\}$
if $\mathsf{Dec}_{\mathsf{sk}}(c'_b) = \mathsf{Dec}_{\mathsf{sk}}(c_0)$	$\xleftarrow{\quad c'_b \quad}$	$c'_b \leftarrow \mathsf{Rand}(c_b, r)$
then $z = 0$ **else** $z = 1$	$\xrightarrow{\quad z \quad}$	**if** $(z = b)$ **then** Accept **else** Reject

Alice (sk, pk, c_0, c_1)		**Bob** (pk, c_0, c_1)
		$r \xleftarrow{\$} \mathcal{R};\ b \xleftarrow{\$} \{0,1\}$
if $\mathsf{Dec}_{\mathsf{sk}}(c'_b) = \mathsf{Dec}_{\mathsf{sk}}(c_0)$	$\xleftarrow{\quad c'_b \quad}$	$c'_b \leftarrow \mathsf{Rand}(c_b, r)$
then $z = 0$ **else** $z = 1$	$\xrightarrow{\quad z \quad}$	**if** $(z = b)$ **then** Accept **else** Reject

Theorem

If the PKE scheme is (computationally) randomizable, then HPINEQ *is complete, computationally sound and perfect HVZK.*

HPEQ

Alice (sk, pk, c_0, c_1)

Bob (pk, c_0, c_1)

HPEQ

Alice (sk, pk, c_0, c_1)

Bob (pk, c_0, c_1)

$r \xleftarrow{\$} \mathcal{R};$

HPEQ

Alice $(\mathsf{sk}, \mathsf{pk}, c_0, c_1)$

Bob (pk, c_0, c_1)

$r \xleftarrow{\$} \mathcal{R}; r_{\mathsf{m}} \xleftarrow{\$} \mathcal{R}_{\mathsf{M}};$

HPEQ

Alice $(\mathsf{sk}, \mathsf{pk}, c_0, c_1)$

Bob (pk, c_0, c_1)

$r \xleftarrow{\$} \mathcal{R}; r_{\mathsf{m}} \xleftarrow{\$} \mathcal{R}_{\mathsf{M}};\ b \xleftarrow{\$} \{0, 1\}$

HPEQ

Alice $(\mathsf{sk}, \mathsf{pk}, c_0, c_1)$

Bob (pk, c_0, c_1)

$r \xleftarrow{\$} \mathcal{R}; r_{\mathrm{m}} \xleftarrow{\$} \mathcal{R}_{\mathrm{M}};\ b \xleftarrow{\$} \{0, 1\}$

$c'_b \leftarrow \mathsf{Rand}(c_b, r)$

HPEQ

Alice $(\mathsf{sk}, \mathsf{pk}, c_0, c_1)$		**Bob** (pk, c_0, c_1)
		$r \xleftarrow{\$} \mathcal{R}; r_\mathsf{m} \xleftarrow{\$} \mathcal{R}_\mathsf{M};\ b \xleftarrow{\$} \{0,1\}$
		$c'_b \leftarrow \mathsf{Rand}(c_b, r)$
$m' \leftarrow \mathsf{Dec}_\mathsf{sk}(c''_b); m \leftarrow \mathsf{Dec}_\mathsf{sk}(c_0)$	$\xleftarrow{\quad c''_b \quad}$	$c''_b \leftarrow \mathsf{MsgRandC}(c'_b, r_\mathsf{m})$

HPEQ

Alice (sk, pk, c_0, c_1)

Bob (pk, c_0, c_1)

Alice		Bob
		$r \xleftarrow{\$} \mathcal{R}; r_{\mathrm{m}} \xleftarrow{\$} \mathcal{R}_{\mathrm{M}};\ b \xleftarrow{\$} \{0,1\}$
		$c'_b \leftarrow \mathsf{Rand}(c_b, r)$
$m' \leftarrow \mathsf{Dec}_{\mathsf{sk}}(c''_b);\ m \leftarrow \mathsf{Dec}_{\mathsf{sk}}(c_0)$	$\xleftarrow{\quad c''_b \quad}$	$c''_b \leftarrow \mathsf{MsgRandC}(c'_b, r_{\mathrm{m}})$
$z \leftarrow \mathsf{MsgRandExt}(m', m)$	$\xrightarrow{\quad z \quad}$	**if** $(z = r_{\mathrm{m}})$ **then** Accept **else** Reject

Alice $(\mathsf{sk}, \mathsf{pk}, c_0, c_1)$		**Bob** (pk, c_0, c_1)
		$r \xleftarrow{\$} \mathcal{R};\ r_{\mathsf{m}} \xleftarrow{\$} \mathcal{R}_{\mathsf{M}};\ b \xleftarrow{\$} \{0,1\}$
		$c'_b \leftarrow \mathsf{Rand}(c_b, r)$
$m' \leftarrow \mathsf{Dec}_{\mathsf{sk}}(c''_b);\ m \leftarrow \mathsf{Dec}_{\mathsf{sk}}(c_0)$	$\xleftarrow{\quad c''_b \quad}$	$c''_b \leftarrow \mathsf{MsgRandC}(c'_b, r_{\mathsf{m}})$
$z \leftarrow \mathsf{MsgRandExt}(m', m)$	$\xrightarrow{\quad z \quad}$	**if** $(z = r_{\mathsf{m}})$ **then** Accept **else** Reject

Theorem

If the PKE scheme is (computationally) randomizable, (computationally) message-randomizable and message-random-extractable, then HPEQ *is complete, computationally sound and perfect HVZK.*

Alice $(r_1, r_2, \mathsf{pk}_1, \mathsf{pk}_2, c_1, c_2)$ — **Bob** $\mathsf{V}(\mathsf{pk}_1, \mathsf{pk}_2, c_1, c_2)$

$r_{\mathsf{m}} \xleftarrow{\$} \mathcal{R}_{\mathsf{M}}$; $(r_1', r_2') \xleftarrow{\$} \mathcal{R}^2$

$r_1'' \leftarrow \mathsf{RandR}(r_1, r_1')$; $r_2'' \leftarrow \mathsf{RandR}(r_2, r_2')$

$c_1' \leftarrow \mathsf{Rand}(c_1, r_1')$; $c_2' \leftarrow \mathsf{Rand}(c_2, r_2')$

$c_1'' \leftarrow \mathsf{MsgRandC}(c_1', r_{\mathsf{m}})$

$c_2'' \leftarrow \mathsf{MsgRandC}(c_2', r_{\mathsf{m}})$ $\xrightarrow{(c_1'', c_2'')}$

$\xleftarrow{b}$ $b \xleftarrow{\$} \{0, 1\}$

if $(b = 0)$ **then** $z = (r_1'', r_2'')$ $\xrightarrow{z}$ **if** $b = 0$ **then return** $(\mathsf{CDec}_{r_1''}(c_1'', \mathsf{pk}_1) = \mathsf{CDec}_{r_2''}(c_2'', \mathsf{pk}_2))$

else $z = (r_1', r_2', r_{\mathsf{m}})$

else $\widetilde{c}_1' \leftarrow \mathsf{Rand}(c_1, r_1')$; $\widetilde{c}_2' \leftarrow \mathsf{Rand}(c_2, r_2')$;

$\widetilde{c}_1'' \leftarrow \mathsf{MsgRandC}(\widetilde{c}_1', r_{\mathsf{m}})$; $\widetilde{c}_2'' \leftarrow \mathsf{MsgRandC}(\widetilde{c}_2', r_{\mathsf{m}})$

return $((\widetilde{c}_1'' = c_1'') \wedge (\widetilde{c}_2'' = c_2''))$

Alice $(r_1, r_2, \mathsf{pk}_1, \mathsf{pk}_2, c_1, c_2)$ — **Bob** $\mathsf{V}(\mathsf{pk}_1, \mathsf{pk}_2, c_1, c_2)$

Alice:

$r_\mathrm{m} \xleftarrow{\$} \mathcal{R}_\mathrm{M}$; $(r'_1, r'_2) \xleftarrow{\$} \mathcal{R}^2$

$r''_1 \leftarrow \mathsf{RandR}(r_1, r'_1)$; $r''_2 \leftarrow \mathsf{RandR}(r_2, r'_2)$

$c'_1 \leftarrow \mathsf{Rand}(c_1, r'_1)$; $c'_2 \leftarrow \mathsf{Rand}(c_2, r'_2)$

$c''_1 \leftarrow \mathsf{MsgRandC}(c'_1, r_\mathrm{m})$

$c''_2 \leftarrow \mathsf{MsgRandC}(c'_2, r_\mathrm{m})$

Alice → Bob: $\xrightarrow{(c''_1, c''_2)}$

Bob: $b \xleftarrow{\$} \{0, 1\}$

Bob → Alice: $\xleftarrow{b}$

Alice:

if $(b = 0)$ **then** $z = (r''_1, r''_2)$

else $z = (r'_1, r'_2, r_\mathrm{m})$

Alice → Bob: $\xrightarrow{z}$

Bob:

if $b = 0$ **then return** $(\mathsf{CDec}_{r''_1}(c''_1, \mathsf{pk}_1) = \mathsf{CDec}_{r''_2}(c''_2, \mathsf{pk}_2))$

else $\widetilde{c}'_1 \leftarrow \mathsf{Rand}(c_1, r'_1)$; $\widetilde{c}'_2 \leftarrow \mathsf{Rand}(c_2, r'_2)$;

$\widetilde{c}''_1 \leftarrow \mathsf{MsgRandC}(\widetilde{c}'_1, r_\mathrm{m})$; $\widetilde{c}''_2 \leftarrow \mathsf{MsgRandC}(\widetilde{c}'_2, r_\mathrm{m})$

return $((\widetilde{c}''_1 = c''_1) \wedge (\widetilde{c}''_2 = c''_2))$

Theorem

If the PKE scheme is perfectly strong randomizable, random-extractable, perfectly message-randomizable and RCD*, then* RSPEQ *is complete, special sound, and perfect zero-knowledge.*

Protocols' Compatibility

Protocols' Compatibility

						Perfect ZK		ZKPoK		
Scheme	Security	RCD	Rand	MsgRand	KeyRand	PEQ	PINEQ	MATCHPEQ	SIGPEQ	RSPEQ
ElGamal [ElG85]	IND-CPA	✓	✓	✓	✓	✓	✓	✓	✓	✓
Paillier [Pai99]	IND-CPA	✓	✓	✓		✓	✓	✓		✓
GM [GM82]	IND-CPA		✓	✓		✓	✓	✓		
DEG [Dam91]	IND-CCA1	✓	✓	✓	✓	✓	✓	✓	✓	✓
CS-lite [CS98]	IND-CCA1	✓	✓	✓		✓	✓			✓
DSCS [PR07]	RCCA	✓	✓				✓			

Agenda

Comparisons for ElGamal

	PET			PIT	
Protocol	[CP93]	PEQ	RSPEQ	[CS03]	PINEQ
Prover	2EXP	6EXP	4EXP	6EXP	6EXP
Verifier	2EXP	4EXP	4EXP	4EXP	4EXP
Rounds	3	4	3	3	4

Comparisons for ElGamal

Protocol	HPEQ	PEQ	HPINEQ	PINEQ	RSPEQ	SIGPEQ
Avg. time (ms)	27.47	70.31	26.13	68.75	62.12	112.98
Deviation	0.21	1.28	0.15	0.6	2.06	3.70

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- Formal definitions for randomizability properties
- Intuitive constructions of zero-knowledge PET-PIT protocols
- Non-interactive variants for sigma protocols via Fiat-Shamir
- Applicable to real-world problems in a “plug & play” manner

Future Work

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- Design non-interactive protocols for plaintext inequality

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- Improve the number of rounds

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- Improve the number of rounds
- Build generic plaintext inequality tests $(<, \leq, \geq, >)$

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Thank you for your time!

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